BIR ZARRALI SHRODINGER OPERATORLARI

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Annotatsiya: Ushbu maqolada markaziy ob'ekt sifatida spektral teorema atrofida qurilgan. Bundan tashqari, chegaralangan operatorlar orqali aylanma yo'lni bosib o'tilmaydi, lekin to'g'ridan-to'g'ri cheklanmagan holatga boriladi. Shuningdek, spektral o'lchovlarning mavjudligi Riesz vakillik teoremasi emas, balki Gerglotz orqali aniqlanadi, chunki bu yondashuv spektral parametr real chiziqqa yaqinlashganda, solventning chegara qiymatlari orqali spektral tiplarni tekshirishga yo'l ochadi.

Kalit so’zlar: Riemann-Lebesgue lemmasi, o'z-o'zidan qo'shilish va spektr, Koshi-Shvars tengsizligi, $RAGE$ teoremasi.

O'z-o'zidan qo'shilish va spektr. d o'lchamdagi bitta zarrachaning Gamiltoniani bilan berilgan

$$H = H_0 + V,$$

bu yerda $V: \mathbb{R}^d \to \mathbb{R}$ zarrachaning potensial energiyasi. Bizni asosan $1 \leq d \leq 3$ holi qiziqtiradi va nisbiy chegaralangan, mos ravishda nisbatan ixcham potentsiallar sinflarini topmoqchimiz. Buning uchun biz H_0 domenidagi funktsiyalarni yaxshiroq tushunishimiz kerak [1].

Lemma 1. Faraz qilaylik, $n \leq 3$ and $\psi \in H^2(\mathbb{R}^n)$. U holda $\psi \in C_\infty(\mathbb{R}^n)$ va har bir $a > 0$ uchun $\| \psi \|_\infty \leq a\|H_0\psi\| + b \| \psi \|$ bo'ladigan $a > 0$ bo'ladi.

Isbot. Muhim kuzatish shundan iboratki $(p^2 + \gamma^2)^{-1} \in L^2(\mathbb{R}^n)$ agar $n \leq 3$. bo'lsa. Demak, $(p^2 + \gamma^2)\hat{\psi} \in L^2(\mathbb{R}^n)$ bo‘lgani uchun Koshi-Shvars tengsizligi

$$\begin{aligned} \| \hat{\psi} \|_1 &= \left\|(p^2 + \gamma^2)^{-1}(p^2 + \gamma^2)\hat{\psi}(p)\right\|_1 \\ &\leq \|(p^2 + \gamma^2)^{-1}\|\|(p^2 + \gamma^2)\hat{\psi}(p)\| \end{aligned}$$

$\hat{\psi} \in L^1(\mathbb{R}^n)$ ni ko'rsatadi. Ammo endi hamma narsa Riemann-Lebesgue lemmasidan kelib chiqadi, ya'ni

$$\begin{aligned} \| \psi \|_\infty &\leq (2\pi)^{-n/2} \|(p^2+\gamma^2)^{-1}\| \left(\|p^2\hat{\psi}(p)\| + \gamma^2 \| \hat{\psi}(p) \|\right) \\ &= (\gamma/2\pi)^{n/2} \|(p^2+1)^{-1}\| (\gamma^{-2}\|H_0\psi\| + \| \psi \|) \end{aligned}$$

isbotni tugatadi. Endi biz birinchi natijamizga keldik [2].

1. teorema. V haqiqiy qiymatli va $n > 3$ bo'lsa $V \in L^\infty_\infty(\mathbb{R}^n)$, $n \leq 3$ bo'lsa $V \in L^\infty_\infty(\mathbb{R}^n) + L^2(\mathbb{R}^n)$ bo'lsin. U holda V H_0 ga nisbatan nisbatan ixchamdir. Xususan, $H = H_0 + V$, $\mathfrak{D}(H) = H^2(\mathbb{R}^n)$, o‘z-o‘zidan qo‘shiladi, pastdan chegaralanadi va

$$\sigma_{ess}(H) = [0, \infty)$$

Bundan tashqari, $C_c^\infty(\mathbb{R}^n)$ H uchun asosdir.

Isbot. Agar $n \leq 3$ bo‘lsa, $V = V_1 + V_2$ va $V_2 \in L^2(\mathbb{R}^n)$ bilan $V_1 \in L^\infty_\infty(\mathbb{R}^n)$ va aks holda $V_2 = 0$ yozing. Aniqrog’i $\mathfrak{D}(H_0) \subseteq \mathfrak{D}(V_1)$ va oldingi lemmamiz shuni ko'rsatadiki, $\mathfrak{D}(H_0) \subseteq \mathfrak{D}(V_2)$ ham o’rinlidir. Bundan tashqari, $f(p) = (p^2 - z)^{-1}, z \in \rho(H_0)$ va $g(x) = V_j(x)$ bilan Lemma 7.21 ga ko’ra ($f \in L^\infty_\infty(\mathbb{R}^n) \cap L^2(\mathbb{R}^n)$ $n \leq 3$ uchun) V_1 ham, V_2 ham nisbatan ixcham ekanligini ko‘rsatadi. Demak, $V = V_1 + V_2$ nisbatan ixchamdir. $C_c^\infty(\mathbb{R}^n)$ Lemma 7.19 bo'yicha H_0 uchun yadro bo'lganligi sababli, Kato-Rellich teoremasi bo'yicha H uchun ham xuddi shunday.

E'tibor bering, $C_c^\infty(\mathbb{R}^n) \subseteq \mathfrak{D}(H_0)$, agar $\mathfrak{D}(H_0) \subseteq \mathfrak{D}(V)$ bo'lsa, bizda $V \in L^2_{loc}(\mathbb{R}^n)$ bo'lishi kerak [3].

Xulosa. V oldingi teoremadagidek bo'lsin. U holda $\Omega \subseteq \mathbb{R}^n$ chegaralangan χ_Ω $H = H_0 + V$ ga nisbatan nisbatan ixchamdir. Xususan, $K_n = \chi_{B_n(0)}$ operatorlari $RAGE$ teoremasining farazlarini qanoatlantiradi.

Vodorod atomi. Biz yadro tomonidan hosil qilingan V tashqi potentsialda harakatlanadigan $\mathbb{R}^3$ dagi bitta elektronning oddiy modelidan boshlaymiz (u boshlang'ichda mustahkamlangan deb hisoblanadi). Agar faqat elektrostatik kuch hisobga olinsa, u holda V Koulomb potentsiali bilan mos keladigan Gamiltonian esa $H^{(1)} = -\Delta - \frac{\gamma}{|x|}$, $\mathfrak{D}\left(H^{(1)}\right) = H^2(\mathbb{R}^3)$ bilan beriladi. Agar potentsial jalb qilinsa, ya'ni

$\gamma > 0$ bo'lsa, u vodorod atomini tavsiflaydi va ehtimol kvant mexanikasidagi eng mashhur modeldir[4].

Biz $\mathfrak{D}\left(H^{(1)}\right) = \mathfrak{D}(H_0) \cap \mathfrak{D}\left(\frac{1}{|x|}\right) = \mathfrak{D}(H_0)$, domenini tanladik va 1-teoremaga asosan $H^{(1)}$ o'z-o'zidan qo'shiladi degan xulosaga keldik. Bundan tashqari, 1-teorema bizga

$$\sigma_{ess}\left(H^{(1)}\right) = [0, \infty)$$

va $H^{(1)}$ pastdan chegaralanganligini aytadi,

$$E_0 = \mathrm{i}\ \ \sigma\left(H^{(1)}\right) > -\infty.$$

Agar $\gamma \leq 0$ bo'lsa, bizda $H^{(1)} \geq 0$ va shuning uchun $E_0 = 0$, lekin agar $\gamma > 0$ bo'lsa, bizda $E_0 < 0$ bo'lishi mumkin va muhim spektrdan pastda ba'zi diskret xos qiymatlar bo'lishi mumkin. $H^{(1)}$ ning xususiy qiymatlari haqida ko‘proq gapirish uchun biz H_0 va $V^{(1)} = -\gamma/|x|$ masshtabga nisbatan oddiy xulq-atvorga ega bo'ling. $U(s)\psi(x) = \mathrm{e}^{-ns/2}\psi(\mathrm{e}^{-s}x)$, $s \in \mathbb{R}$ kengayish guruhini ko'rib chiqaylik, bu kuchli uzluksiz bir parametrli unitar guruhdir. Generatorni osongina hisoblash mumkin:

$$D\psi(x) = \frac{1}{2}(xp + px)\psi(x) = \left(xp - \frac{\mathrm{i}n}{2}\right)\psi(x),\ \psi \in \mathcal{S}(\mathbb{R}^n)$$

Endi $U(s)$ ning $H^{(1)}$ dagi harakatini tekshiramiz:

$$H^{(1)}(s) = U(-s)H^{(1)}U(s) = \mathrm{e}^{-2s}H_0 + \mathrm{e}^{-s}V^{(1)},\ \mathfrak{D}\left(H^{(1)}(s)\right) = \mathfrak{D}\left(H^{(1)}\right)$$

Endi $H^{(1)}\psi = \lambda\psi$ deylik. Shunda $\left\langle\psi, \left[U(s), H^{(1)}\right]\psi\right\rangle = \langle U(-s)\psi, \lambda\psi\rangle - \langle\lambda\psi, U(s)\psi\rangle = 0$ va demak,

$$\begin{aligned} 0 \quad &= \lim_{s\to 0}\frac{1}{s}\left\langle\psi, \left[U(s), H^{(1)}\right]\psi\right\rangle = \lim_{s\to 0}\left\langle U(-s)\psi, \frac{H^{(1)} - H^{(1)}(s)}{s}\psi\right\rangle \\ &= -\left\langle\psi, \left(2H_0 + V^{(1)}\right)\psi\right\rangle \end{aligned}$$

Shunday qilib, biz virial teoremani isbotladik.

Foydalanilgan adabiyotlar ro'yhati

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[3] W. O. Amrein, Non-Relativistic Quantum Dynamics, D. Reidel, Dordrecht, 1981.
[4] W. O. Amrein, A. M. Hinz, and D. B. Pearson, Sturm-Liouville Theory: Past and Present, Birkhauser, Basel, 2005.